PROBLEMS OF DEVELOPMENT OF GAS, GAS CONDENSATE AND OIL/GAS/CONDENSATE FIELDS

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A new filtration equation considering the impact of connate water and one-side open pores on gas movement in productive formation

At present, for studying gas filtration in productive formations of gas fields, a mathematical model of homogeneous compressed liquid movement in porous media is used. If there is no connate water in this media, application of this model would be quite justified. Actually, in case of reservoir wettability, free water buried in porous medium and occupying narrow passages between rock grains destroys homogeneity of fluid medium (gas), thus creating an absolutely different – as compared to homogeneous liquid – picture of gas filtration.

Connate water takes 30 to 70 % of vapor volume and isolates gas bubbles located in larger pores from each other. To cause gas movement in a layer, it is necessary to push out water from narrowing channels (constrictions, cross-flows) connecting large pores, and create a pressure difference between entrapped therein bubbles. The size of this pressure difference depends on cross-sections of channels occupied by water and gas bubbles, and on inter-phase tension at ‘gas – water’, ‘water – rock’, ‘gas – rock’ contacts. Further, to make long story short, cross-sectional dimensions will be called ‘average values’ (or in simple terms – ‘diameters’).

Thus, gas-bearing productive formation can be represented as a porous medium, in large pores of which there are gas bubbles divided among themselves by water valves that open only at certain pressure differences and return under the influence of capillary forces to their initial position at gas pressure balance in neighboring pores separated by such ‘valve’. Before trying to estimate the values of these differences, it is necessary to make some remarks concerning physical modeling of gas filtration processes through water-saturated core, which is carried out in laboratories studying the physics of formations.

First, in the laboratory, it is impossible to create conditions of gas bubbles distribution in the porous medium with connate water. For geological period of time, in view of gas diffusion, concentration of gas molecules in all bubbles should become equal. Equality of concentrations corresponds to equality of pressures inside the bubbles. But equal pressures in gas bubbles located at the same depths in liquid, which is under the same head pressure and under the same temperature, can occur only, if such bubbles have same diameters. It means, that geological time made distribution of gas bubbles in porous medium more homogeneous. Therefore, after core extraction and drying, all subsequent manipulations with repeated saturation by water and gas will be rather far from reality of untouched productive formation.

Second, again – because of geological time, it is impossible to simulate the degree of rock hydrophilic and hydrophobic properties in the examined core. All hydrocarbons, including methane, can for a long time make surfaces of quartz and calcite crystals more ‘hydrophilic’.

Development of Cenomanian deposits, especially at its initial stage, makes it possible to assume that productive formation rock in these deposits is hydrophilic. Otherwise, water located in porous medium in the form of drops at high depressions should be pushed out in

Keywords: gas filtration model, medium permeability, pressure gradient, reservoir hydrophilic behavior, connate water, one-side open pores, flowing pores.
considerable volumes together with gas from the very beginning of deposits development, which we do not observe in real practice.

Gas filtration mechanism, at presence of free connate water in the reservoir, is greatly influenced by two types of pores: flow-through and one-side open. Porous medium is represented schematically on Fig. 1.

The volume ratio ($\Psi$) of one-side open pores ($\Omega_{\text{one-sided}}$) to the general porosity ($\Omega$) depends on permeability of productive rock (Fig. 2); all experimental points without intervals and deviations are densely concentrated on around the curve.

Fig. 2 shows, that in low-permeability reservoirs at average permeability of 10 mD, relative content of one-side open pores is 0.8, i.e. only 20 % of them are the flow-thrgh pores; at permeability $\approx$ 1000 mD, 80 % are flow-through pores; at permeability 100 mD, volumes of flow-through and one-side open pores are about the same.

Average permeability of Cenomanian deposits reservoirs is 600 mD, therefore one-side open pores according to the graph (see fig. 2) will be 0.3 of the total amount of all open pores (one-side open + flow-through pores). The share of 0.3 volume of one-side open pores from the total amount of discovered Cenomanian deposits is confirmed by long-term monitoring of residual gas saturation factors in water-flooded zone of productive formation ($\approx$ 0.3 remaining a constant value after three-to-four years from the wetting time at gradual pressure drop).

With decrease of pressure in the wetting zone the mass of gas ($M$) is decreasing, while the volume of entrapped gas defined by one-side open pores remains constant. So, it would be impossible to extract all gas from formation, even by creating vacuum in the mouth of the bore hole. In the formation, at its full flooding, there remains:

$$M \approx \rho \Omega_{\text{dep}} \Psi,$$

where $\rho$ – is gas density at average capillary pressure created by water stoppers isolating one-side open pores from flow-through pores; $\Omega_{\text{dep}}$ – initial gas-saturated volume of the deposit.

For low-permeability reservoirs, e.g. with average permeability of 10 mD, $\Psi = 0.815$. In such reservoir, at open porosity of 0.15, one-side open porosity will be 0.122, and flow-through porosity – 0.028 (i.e. 1/5 of all open porosity).

At transition to development of deep deposits, the number of productive reservoirs with low permeability will increase. Hence, in such productive formations relative gas stocks in one-side open pores will increase, and the

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**Fig. 1.** Arrangement of connate fluid water and gas in porous space at the absence of pressure gradient: 1, 2, 3, 4, 5 – flow-through pores; 6, 7, 8, 9 – one-sided pores (pores 1–9 belong to the group of open pores); 10 – a closed pore isolated from filtration process.
conditions of their extraction will become more complicated. In reference [1], it is pointed out that ‘in the complex structure reservoirs with secondary porosity, volumes of one-side open pores can be of considerable values even at increased permeability’. High share of one-side open pores should to a great extent affect the gas filtration mechanism. One should keep in mind that one-side open pores do not filter gas through themselves. At certain pressure drop between one-side-open and open pores, the first is the internal source of additional gas mass arriving to the second one. On fig. 3, it is shown that this source is effective during the whole time of productive formation exhaustion, periodically opening and closing fluid water stopper located in the narrow passage between one-side open and flow-through pores.

One should keep in mind that all hydrophilic rock surface is covered with immovable water film 0.5 microns thick, by its properties differing from free water (see fig. 3). If the formation gas contains condensate, pore walls over a water film are covered with hydrocarbons films (from the heaviest to the lightest ones). One can assume that these films are not continuous, and free water can partially contact with non-fluid water film. However, in most cases the contact angle of wetting for free water in rock shall be over zero degrees. If, in the course of filtration, pressure $P_2$ in the pore 2 is less than pressure $P_1$ in the pore 1 (see fig. 3), water stopper 3 shall move towards pore 2 spreading by its walls $3^*$ with the subsequent formation of a bubble 4, which then will burst, so that a part of gas from pore 1 will be ‘thrown’ to pore 2 before pressure levels in both pores. After that, under the influence of surface (capillary) forces, stopper 3 will return to its initial position. A new pressure drop will be required in the open pore 2 for emission of a new gas portion from one-side open pore 1 to pore 2.

To push water stopper from a narrow cross-flow of a larger diameter (fig. 4) the following pressure difference will be required:

$$\Delta P = 2\sigma \cos \varphi \left( \frac{1}{r} - \frac{1}{R} \right),$$

where $r$ – is the average cross-flow radius; $R$ – radius of a larger pore; $\sigma$ – water surface tension at the borderline of its contact with gas.

A gas bubble formed at gas pushing out, which at its destruction will either open the exit to gas from one-side open pore or will burst at $\Delta P_1 = \frac{4\sigma}{R}$, or will come off the end of the narrowed channel and will also burst at $\Delta P_2 = \frac{4\sigma}{R^2}$ (see fig. 3).

Using above formulas, it is possible to estimate $\Delta P$ and $\Delta P_1$, at which bubbles will be pushed out and burst. Similar calculations have been received for porous terrigenous reservoirs of Lower Cretaceous sandstones at the Eastern

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**Fig. 2. Relative content of one-side open pores $\Psi$ vs gas permeability factor $K_{\text{perm}}$**

(B.I. Tulbovich [1])

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$$\Psi = \frac{\Omega_{\text{one-side}}}{\Omega}$$
Pre-Caucasian region [2], carbonate reservoirs of Orenburg gas condensate fields (GCF) [3], and Cenomanian deposits of productive formations in the north of Western Siberia. Dimensions of pores and channels between them (authors of [2] call them ‘constrictions’) were defined using grain-size distribution of cores.

The average radius $R$ of extensions for a large number of cores from terrigenous reservoirs of Lower Cretaceous sandstones of the Eastern Pre-Caucasian region proved to be of 20 microns, and radius of constrictions $r$ – of 8 microns; length of constrictions varied between 100 and 200 microns. Such relationships have been defined by roundstone quartz crystals at their dense packing.

Dimensions of pores and constrictions in carbonate reservoirs of Orenburg GCF are much smaller: prevailing pores are those with $R = 1–3$ microns, and pores connected with channels – with $r = 0.05–0.15$ microns. On fig. 5 you can see micro-photos of porous differences for limestone of the Orenburg GCF productive formation obtained by N.G. Kulikova by means of scanning electronic microscope.

For Cenomanian sandstones, $\Delta P$ and $\Delta P_1$ have been estimated for conditionally assumed dimensions of pores with $R = 10$ microns, $r = 2–3$ microns. Water surface tension at the boundary with gas can be termed as not depending on average pore radii given above, but it will significantly depend upon the medium temperature and pressure of gas contacting with it.

For water at temperature of 30 °C and gas pressure 100·10^5 Pa, surface tension will be 52·10^{-3} N/m [4]; wetting angle in calculations is assumed as equal to 45° ($\cos 45° = 0.7071$).

Calculated values of $\Delta P$ for pushing water stoppers from narrow channels connecting pores at specified above average dimensions were:

- for Pre-Caucasian region reservoirs – of $0.055·10^5$ Pa;
• for Cenomanian productive deposits – of $0.17 \times 10^5$ Pa;
• for carbonate rocks of Orenburg GCF – of $4.6 \times 10^5$ Pa.

If we refer these differences to lengths of narrowing channels combined with dimensions of extensions equal to $2R$, we will receive pressure gradients of some tens and hundreds atmospheres per meter. It is clear that such gradients are impossible at working out gas fields, if we take into account that depressions (e.g., in boreholes exploiting Cenomanian fields) do not exceed 1.5–2 atmospheres in the beginning of exploration works.

Hence, at all gas deposits filtration occurs by consecutive ingress of additional gas volumes in the process of gas transfer to the borehole, which are filling pores of productive rock from distances more and more remote from the axis of the operating borehole.

For a better understanding of occurring at that processes, we can use the analogy, which in no way describes gas filtration process in porous medium, but shows the sequence of processes, though of a physically different nature. Locomotive cannot put in motion the whole freight train at once: its engines cannot develop a force exceeding the friction force of the train at rest. Therefore, it has to move cars from rest one by one. To do it, the operator drives the back to compress buffers (springs) between first cars, and then begins to move them forward, one by one, involving first cars in movement.

To promote the inflow of gas into borehole, we should decrease pressure at its bottom. At that, the created depression always is higher than $\Delta P$ required for pushing out liquid stoppers between pores filled with gas. Therefore, gas begins to leak into the borehole from adjoining cylindrical part of productive formation. In ideal case, the limiting area radius in the form of a cylindrical surface in productive formation stays motionless till $\Delta P$ in water stoppers of narrow channels connecting open pores proves sufficient for unblocking the neighbouring and more remote from radius pores. Average $\Delta P$ values required for pushing out water stoppers between the open pores should be same as those for stoppers between one-side open and flow-through pores.

In view of variety of channels dimensions between flow-through pores of $\Delta P$ value for pushing out connate fluid water filling these channels, these belong to different pressure intervals in productive formations. In mathematical models, we have to use their average values.

For further argumentation, we will need to estimate pressure gradients for models used currently for solving problems of gas filtration in porous medium.

If for filtration equation we use the state equation

\[ \rho = AP, \]  

Fig. 5. Porous differences of the Orenburg GCF productive formation limestone:

a – image received using scanning electronic microscope at 4360-times magnification;
b – fragment of fig. 5a at 40000-times magnification
where \( A = \frac{\rho_0 z_0 T_0}{P_0 z(P,T)} \) (index in the form of asterisk (*) refers to the parameter at standard conditions; \( z \) – real gas compressibility factor; \( T \) – absolute gas temperature), then the pressure gradient shall be calculated by formula

\[
\frac{dP}{dr} = \frac{D_1}{2} \frac{1}{r},
\]

(2)

where \( D_1 \) is a constant.

If for the same purpose the following equation

\[
\rho = C e^{C_2},
\]

(3)

is used, where \( C \) and \( C_2 \) – are constants, we can get a linear filtration equation using quotient derivatives of parabolic type, which is incomparably easier to solve than equation (1).

In [5], it is shown that formal approximation of state equation by formula (3) at certain ranges in changes formation pressure at deposit development results in relatively small errors as compared to the density estimated by formula (1). The advantage of using formula (3) also provides a possibility to estimate the error, while at using formula (1) for nonlinear equation we have to linearize it for analytical solutions without a possibility of obtaining potential errors.

At using equation (3), pressure gradient will be equal to

\[
\frac{dP}{dr} = D_2 \frac{1}{r},
\]

(4)

where \( D_2 \) – is a constant.

At relevant selection of \( C_2 \) in exponential formula of state equation, the debit of the borehole will totally coincide with the debit obtained by the filtration equation relative to the pressure square. Then we should check how pressure gradients estimated by formulas (2) and (4) would differ.

As an example, we have summed up in the table results of calculating pressure gradients by formulas (2) and (4) for different distances from borehole axis at stabilized gas filtration mode (pressure at gas engress loop \( P_{\text{loop}} = 100 \cdot 10^5 \) Pa and at the borehole wall \( P_{\text{bh}} = 98 \cdot 10^5 \) Pa; loop radius \( R_{\text{loop}} = 500 \) m and that of the borehole \( r_{\text{bh}} = 0,05 \)).

According to GOST 26450.2-85 [6], permeability measurement is regulated as follows: ‘Carry out measurements at pressure after the sample equal atmospheric one … Carry out 3-times measurement of gas flow rate through the sample at various pressure differences within \((1 \cdot 10^{-3})–(3 \cdot 10^{-1}) \) MPa.’ In cores 30 mm long, interval of gradients at specified pressure differences will be equal \((0,033–10) \) MPa/m. In the range of such gradients, tests have been carried out as stated in the book [7].

At comparing gradients of the standard [6] with gradients in the table we can see, that the minimum gradient value in the range recommended by the standard corresponds to the calculated gradient at a distance of several tens meters from the borehole axis. In this example (see the table), this distance \((r)\) equals to 21,7 m. At other distances to the end of gas supply zone \((R_{\text{loop}} = 500 \) m) calculated gradients are dozen times lower than those created in the laboratory at studying core permeability.

One can assume that so small pressure gradients at gas filtration in productive formations (obtained at using existing models) simply do not exist in reality. Laboratories do not study permeability of water-saturated cores at pressure gradients of several water column millimeters per meter. Easier and more justified would be to represent the process of gas offtake from productive formation as an extension of the offtake zone with external fluid pressure.

### Pressure gradients calculated by formulas (2) and (4) for different distances from borehole

<table>
<thead>
<tr>
<th>( r, \text{m} )</th>
<th>Pressure gradient ( \frac{dP}{dr}, \text{Pa/m by formula (2)} )</th>
<th>Pressure gradient ( \frac{dP}{dr}, \text{atm/m by formula (4)} )</th>
<th>( \text{mm of water/m} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 2,17 \cdot 10^4 )</td>
<td>( 2,17 \cdot 10^4 )</td>
<td>2140</td>
</tr>
<tr>
<td>10</td>
<td>( 2,17 \cdot 10^1 )</td>
<td>( 2,17 \cdot 10^3 )</td>
<td>214</td>
</tr>
<tr>
<td>21,7</td>
<td>( 1,0 \cdot 10^1 )</td>
<td>( 1,0 \cdot 10^1 )</td>
<td>98,7</td>
</tr>
<tr>
<td>100</td>
<td>( 2,16 \cdot 10^3 )</td>
<td>( 2,16 \cdot 10^2 )</td>
<td>21,4</td>
</tr>
<tr>
<td>200</td>
<td>( 1,08 \cdot 10^3 )</td>
<td>( 1,08 \cdot 10^2 )</td>
<td>10,7</td>
</tr>
<tr>
<td>300</td>
<td>( 7,18 \cdot 10^2 )</td>
<td>( 7,18 \cdot 10^1 )</td>
<td>7,14</td>
</tr>
<tr>
<td>400</td>
<td>( 5,38 \cdot 10^1 )</td>
<td>( 5,38 \cdot 10^0 )</td>
<td>5,36</td>
</tr>
<tr>
<td>500</td>
<td>( 4,38 \cdot 10^1 )</td>
<td>( 4,38 \cdot 10^0 )</td>
<td>4,28</td>
</tr>
</tbody>
</table>
boundary, and at constant pressure gradient thereof equal to the initial value (less than the gradient at borehole wall, but coming closer to it by value).

At constant borehole debit, the volume of gas offtake zone will be proportional to the product of its square fluid radius \( r \) and formation thickness, and the increase of the radius will be proportional to the square root of operation time. Hence, frequently used representation of some nonexistent ‘offtake zone’ for each particular borehole is not so far from reality.

If in porous medium unit volume share of flow-through pores is \( \varphi_1 \) (flow-through porosity), and the share of one-side open pores – \( \varphi_2 \) (one-side open porosity), the sum of these shares will be equal to open porosity \( \varphi \). In good approximation, considering geological history of filling of trap with gas, the average initial gas saturation \( (\alpha) \) can be considered identical both for flow-through and for one-side open pores. Hence, the initial volumes of gas in flow-through and one-side open pores per volume unit of porous medium will be equal to \( \varphi_1 \alpha \) and \( \varphi_2 \alpha \), correspondingly.

If, as a result of gas field operation, pressure in flow-through pore bubbles (flow-through pressures) is lowered, then at certain differences between these flow-through pressures and pressures in bubbles of neighboring one-side open pores (one-side open pressures) will open water stoppers connecting these pores, and part of gas from one-side open pores will flow into flow-through pores. Then these water stoppers under the influence of surface forces at ‘gas – water’ borderline will be closed, and their next opening will come after occurrence of the same pressure difference as in the previous case. Such step-by-step supply of subsequent gas portions from one-side open pores to flow-through pores will occur during all the whole period of field development.

Each individual one-side open pore will work as a pulse source of gas. It can be shown that at achieving a certain (same by value) difference between the pressures in this pore and in the connected to it flow-through pore, the latter will receive the same mass of gas irrespective of the absolute pressure in the one-side open pore. Different one-side open pores have different dimensions of flow-through channels into connected with them flow-through pores. Therefore, for pushing out water stoppers and ejecting portions of gas from one-side open pores into flow-through pores different \( \Delta P \) values are required. However, for each of ‘one-side open – flow-through pores’ pairs the following statement would be fair: if formation pressure is changing according to the linear law, the mass of gas extracted from the one-side open pore to the in flow-through pore will be time proportional.

The sum of the emissions created by the whole variety of one-side open pores in some allocated volume of porous medium will form a distributed source of additional mass of gas coming into flow-through pores (i.e., in the filtered gas flow). Strictly speaking, pressure differences required for ‘squeezing’ of water stoppers between the pores will depend of formation pressure \( (P_f) \) and will increase with its dropping. As a matter of fact, water surface tension factor at the borderline with gas depends on pressure. E.g., tentatively – for gas conditions of Cenomanian deposits – the value of water surface tension at the borderline with gas and at initial \( P_f = 12 \text{ MPa} \) was 3/4, and at \( P_f = 2 \text{ MPa} \) constituted 0.95 of tension under standard conditions.

It is not difficult to estimate the mass of gas emitted in elementary volume \( d\Omega \) per second, e.g., from one-side open pores of Cenomanian formations. Gas mass \( (M) \) in one-side open pores of 1 m³ porous rock at initial pressure of 120 MPa is 5,134 kg. Let us assume that the basic development period is 20 years = 6,32·10⁶ s. This means that from 1 m³ of productive rock with one-side open pores in a second we can extract \((5,134/6,32)\cdot 10^4 \text{ kg/s}, \) and from elementary volume \( d\Omega – 0.82\cdot 10^4 d\Omega \text{ kg/s}. \)

For each neighboring pair of ‘one-side open – flow-through’ pores, \( \Delta P \) at which every next emission will slowly increase with pressure drop, since this will increase water surface tension at the borderline with gas.

As it has been pointed out before, for different pairs \( \Delta P \) should differ, as it depends on the dimensions of passages between one-side open and flow-through pores. We have all grounds to assume, that for rocks of the same productive formation dimensions of the larger number of these passages will be grouped around some average value or the range of dimensions specified by specialists investigating core material.

Hence:

\[
\frac{\Delta P_{av}}{L_{av} + D_{f,av}} = \left(\frac{dP}{dr}\right)_{av},
\]

where \( \Delta P_{av} \) – is the average pressure difference resulting in the next gas emission; \( L_{av} \) – average length of the channel connecting neighboring of one-side open and flow-through pores;
$D_{p,av}$ – average diameter of pores; \( \frac{dP}{dr} \) – initial pressure gradient which always occurs in productive formation at gas filtration.

This conclusion, at first sight, seems wrong: what do non-filtering one-side open pores have to do with initial gradient? The fact is that one-side open and flow-through pores should not differ by their average dimensions of connecting channels (constriction) as both of them serve as passages between crystals forming the porous rock, and only a part of them is clogged with cementing inclusions. For this reason, pressure drops required for pushing out water stoppers should be the same both between ‘one-side open – flow-through’ and ‘flow-through – flow-through’ pairs of pores.

Mechanism describing the ‘work’ of one-side open pores as internal gas sources can provoke the following objections. First, at pushing out water stopper from constriction connecting one-side open pores with flow-through ones (see fig. 3) water will spread over the walls of flow-through pore reducing its volume. However, at that the total volume of one-side open and flow-through pores does not change, thus facilitating the calculation of average pressure after gas emission (if there is enough time for pressure balancing). Another objection is that gas is filtrated at its emission from one-side open pores (either in the form of ‘jumps’ as emission from a one-side open pore, or by ‘jets’ at high pressure gradients). Nevertheless, in both cases for emission from a one-side open pore, $\Delta P$ between pressure in one-side open and flow-through pores should be sufficient to push out water stopper between them.

At development of large gas fields, falling of average formation pressure is practically almost linear. But at linear pressure changes in flow-through pores, gas emissions from one-side open pores will occur in regular time intervals. As at each next emission the same mass of gas will be emitted to the flow-through pore, then in a certain unit of time the internal source in porous medium represented by one-side open pores will work with constant ‘productivity’. If all narrowing channels between one-side open and flow-through pores were identical, such source would ‘work’ in a pulse mode. But channels connecting these pores are of a wide range of dimensions, and hence there is a wide interval of pressure differences at which a one-side open pores will open: for each dimension exists its own pressure difference with its own emission frequency. Superimposing over each other, these emissions will level gas supplies to flow-through pores with time, and at linear changes of average formation pressure, in aggregate, act as a source with a constant mass flow in volumetric unit of porous medium.

Flow-through pores in gas-saturated formation are not continuous channels free from connate water. They are separated in narrow channels by water stoppers which open at creation of certain pressure differences of filtered gas between neighboring flow-through pores.

In this connection, at defining gas permeability of water-saturated cores, there should be an interval of pressure drops from a certain value, at which filtration begins, to the pressure drop, at which the Darcy law becomes applicable, i.e. permeability becomes constant. Normally, this transit area is not registered at defining gas permeability of water-saturated cores, as more often high gradients existing near the borehole trunk are investigated. However, filtration in other productive formation spaces occurs at much smaller gradients. Hence, permeability in these conditions would always depend on pressure gradient.

Extensive experimental material about the effect of water saturation on permeability is reflected in [7]. However, dependences given there have been also received at high water saturation values. It seems that there is no experimental data obtained at small gradients and low water saturations values.

Taking into account the above, permeability factor of water-saturated core ($k$) depending upon pressure gradient can be approximated by function

$$k = k_0 f\left(\frac{dP}{dr} \cdot \frac{dp}{dr}\right),$$

where $k_0$ – constant permeability factor at gas filtration according to Darcy law; $f\left(\frac{dP}{dr} \cdot \frac{dp}{dr}\right)$ – the dimensionless function depending on dimensionless gradient \( \frac{dP}{dr} \cdot \frac{dp}{dr} \).
gradient at radial filtration; \( \frac{dp}{dr} \) – assumed as a unit of measurement).

Then, we can introduce the designation

\[
\left( \frac{dP}{dr} \bigg/ \frac{dp}{dr} \right) = \xi
\]

Approximate graphic representation of \( f(\xi) \) function variation from 0 to 1 in terms of \( \xi \) variation (from 0 to the gradient at which Darcy law stops to be effective) is given on fig. 6. The variation law can be approximated by different smooth and non-smooth functions. In this article, we use the infinitely differentiated function

\[
f(\xi) = \frac{1}{1 + ae^{-\xi/b}},
\]

with two positive arbitrary dimensionless constants \( a \) and \( b \), allowing in the range \( 0 < \xi < \xi_1 \) to bring the value of this function close to 0, thus approximating initial gradient in point \( \xi_1 \), and the rate of experimentally obtained gradient between \( \xi_1 \) and \( \xi_2 \) – up to gradient value, above which permeability will not depend on it.

In other words, when using formula (5) we can shift points \( \xi_1 \) and \( \xi_2 \) along axis \( \xi \) in any way, providing at that any approximation of function \( f(\xi) \) to zero in the point \( \xi_1 \) and to “one” in the point \( \xi_2 \).

The filtration equation shall be derived for cylindrical system of co-ordinates \((z, r, \varphi)\). Axis \( OZ \) coincides with borehole axis. The mass of gas emitted through cylindrical surface \( A D D_1 A_1 \) (fig. 7) during a \( dt \) period will be equal to:

\[
M_{ cyl} = \rho V r d\varphi d z dt,
\]

where \( \rho \) – gas density; \( V \) – radial gas filtration speed.

During the same time, through cylindrical surface \( BCC_1 B_1 \) will be emitted a mass equal to

\[
\left( \rho + \frac{\partial \rho}{\partial r} dr \right) \left( V + \frac{\partial V}{\partial r} (r + dr) \right) d\varphi d z dt =
\]

\[
= \rho V r d\varphi d z dr + r \frac{\partial}{\partial r} (\rho V) d r d\varphi d z dt +
+ \rho V d\varphi d z d r dr dt.
\]

At drawing formula (7), we excluded summands which included infinitely small values higher than \( dr \).

Change of gas mass in the \( ABCDA_1 B_1 C_1 D_1 \) prism due to radial gas ingress and egress will be equal to the difference between ingress and egress masses:

\[
dM_{ prism} = -\frac{\partial}{\partial r} (\rho V) r d r d\varphi d z d t - \rho V d r d\varphi d z d t.
\]

To the change of gas mass calculated for curvilinear parallelepiped (8), we should add \( M \), emitted from one-side open pores into flow-through ones. Separated from flow-through pores by water stoppers, these pores perform a function of uniformly distributed internal source of gas. At above conditions, mass of gas \( M \) is proportional to product of a constant \( G \) factor (\( G \) has the dimensions of time-divided density in an elementary curvilinear parallelepiped volume \( \Omega = rd\varphi d z d r d t \)) and \( dt \).

On the other hand, at \( \Omega \) itself being constant, the calculated change of gas mass due to gas filtration through this volume and its internal sources from one-side open pores will result in change of its density. If in the beginning gas density was equal to \( \rho \), after \( dt \) period it will be equal to \( \rho + \frac{\partial \rho}{\partial t} dt \). The initial mass of gas \( M = \rho d\Omega = \rho \varphi_1 r d\varphi d z d r d t \), and in a certain period of time its value will become equal to \( \left( \rho + \frac{\partial \rho}{\partial t} dt \right) \varphi_1 r d\varphi d z d r \). Hence, the change of mass in volume \( \Omega \) in the \( dt \) period of time will be equal to \( \varphi_1 \alpha \frac{\partial \rho}{\partial t} r d\varphi d z d r \).

In the last formula \( \varphi_1 \) is the flow-through porosity factor. The one-side open porosity factor (\( \varphi_2 \)) is included as a multiplier in factor \( G \). Comparing obtained masses and reducing both parts of equality by \( rd\varphi d z d r d t \), we get a continuity equation

\[
-\frac{\partial}{\partial r} (\rho V) - \frac{1}{r} \rho V + G = \varphi_1 \alpha \frac{\partial \rho}{\partial t},
\]

which can be re-written in the following form:

\[
-\frac{1}{r} \frac{\partial}{\partial r} [r(\rho V)] + G = \varphi_1 \alpha \frac{\partial \rho}{\partial t}.
\]
Fig. 6. Function $f(\xi)$ envelope

Fig. 7. Elementary curvilinear parallelepiped
Connection between $V$ and pressure gradient is established by Darcy law:

$$V = \frac{k \frac{\partial \rho}{\partial r}}{\mu}. \quad (11)$$

Permeability of water-saturated core depends on pressure gradient at its small values.

Further, the reduced dependence (5) considering experimental results is used:

$$k = \frac{k_0}{1 + a \exp \left( -b \frac{\partial P}{\partial r} \frac{\partial \rho}{\partial r} \right)}. \quad (12)$$

Connection between gas density and pressure is established by ‘pressure-volume’ equations (1) and (3).

Equation (1) is used to replace density by pressure, and (3) – for linearization of the equation. From (1) we get a ratio

$$\frac{\partial \rho}{\partial r} = \frac{1}{\rho} \frac{\partial \rho}{\partial r}. \quad (13)$$

Using formulas (11) – (13), we obtain the final equation for gas filtration in water-saturated reservoir:

$$\frac{\partial P}{\partial t} = \frac{k_0}{\mu \lambda \alpha \phi} r \frac{\partial P}{\partial r} \left[ \frac{\partial P}{\partial r} \frac{\partial P}{\partial r} \right] + \frac{G}{a \phi A} \left( -b \frac{\partial P}{\partial r} \frac{\partial \rho}{\partial r} \right) + \frac{\partial P}{\partial r}. \quad (14)$$

At small pressure gradients and low permeability, the obtained equation should reflect the real gas filtration process closer than currently used equations which are suitable for description of the process only near the extracting borehole.

Low permeability means the presence of a large volume of one-side open pores approaching the volume of flow-through pores, which involves a rather large second summand in the right part of equation (14), while small pressure gradients, as it has been pointed out, are inherent for larger areas of developed fields. In these zones, permeability factor will significantly depend on the pressure gradient.

References


